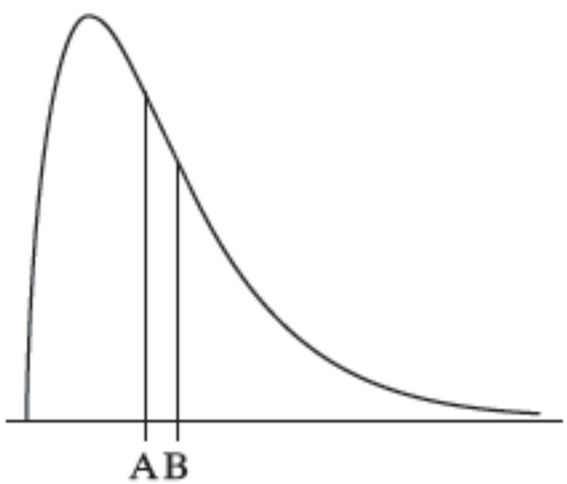
Chapter Review Exercises (page 136)

R2.1 (a) $z = \dfrac{179-170}{7.5} = 1.20$. Paul's height is 1.20 standard deviations above the average male height for his age.
(b) 85% of boys Paul's age are shorter than Paul.

R2.2 (a) Reading up from 7 hours on the *x*-axis to the graphed line and then across to the *y*-axis, we see that 7 hours corresponds to about the 58th percentile.
(b) To find Q_1, start at 25 on the *y*-axis, move across to the line and down to the *x*-axis. Q_1 is approximately 2.5 hours. To find Q_3, start at 75 on the *y*-axis, move across to the line and down to the *x*-axis. Q_3 is approximately 11 hours. Thus, $IQR = 11 - 2.5 = 8.5$ hours per week.

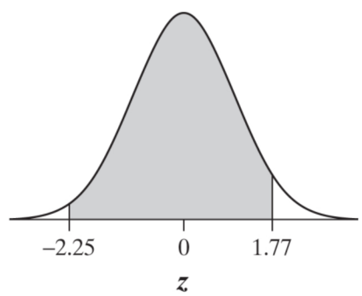
R2.3 (a) If we converted the guesses from feet to meters the shape of the distribution would not change. The new mean would be $\dfrac{43.7}{3.28} = 13.32$ meters, the median would be $\dfrac{42}{3.28} = 12.80$ meters, the standard deviation would be $\dfrac{12.5}{3.28} = 3.81$ meters, and the *IQR* would be $\dfrac{12.5}{3.28} = 3.81$ meters.
(b) The mean error would be $43.7 - 42.6 = 1.1$ feet. The standard deviation of the errors would be the same as the standard deviation of the guesses, 12.5 feet, because subtracting a constant from each observation in a distribution does not change the spread.

R2.4 (a) Answers will vary but the line indicating the median (line A in the graph below) should be slightly to the right of the main peak with half of the area to the left and half to the right.
(b) Answers will vary but the line indicating the mean (line B in the graph below) should be slightly to the right of the line for the median at the balance point.

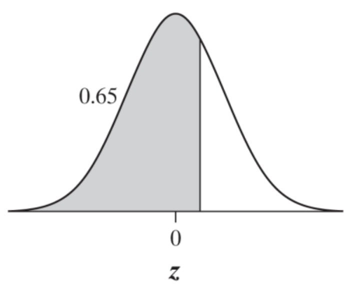


2.5 (a) In a Normal distribution, 99.7% of the values fall within 3 standard deviations of the mean. Thus, 99.7% of horse pregnancies will last between 336 – 3(3) = 327 days and 336 + 3(3) = 345 days.
(b) 339 is one standard deviation above the mean and 68% of observations are within one standard deviation of the mean, so $\dfrac{100\% - 68\%}{2} = 16\%$ of the horse pregnancies last longer than 339 days.

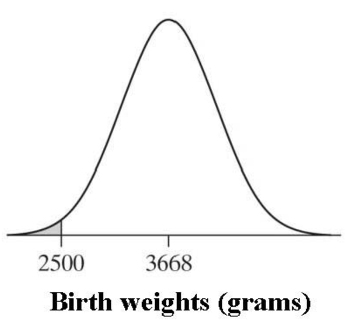
R2.6 (a) The graph is given below. Subtract the area to the left of $z = -2.25$ from the area to the left of $z = 1.77$ to get $0.9616 - 0.0122 = 0.9494$. The proportion of observations between -2.25 and 1.77 is 0.9494. *Using technology*: The command normalcdf(lower: –2.25, upper: 1.77, μ : 0, σ : 1) gives an area of 0.9494.



(b) If 35% of all values are greater than a particular *z*-value, then 65% are lower (see graph below). Looking up 0.65 in the body of Table A, *z*-score of 0.39 gives the closest value (0.6517). Using technology: The command invNorm(area: 0.65, μ : 0, σ : 1) gives $z = 0.385$.

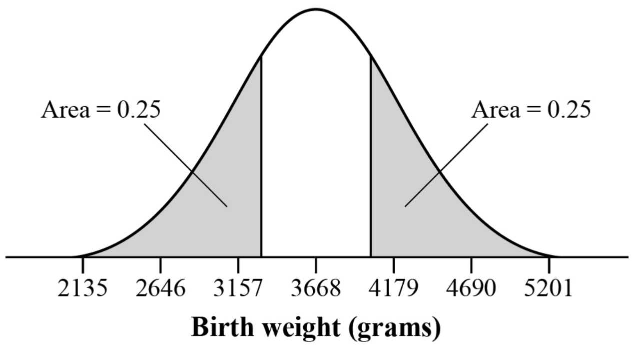


R2.7 (a) **Step 1: State the distribution and values of interest.** Birth weights follow a Normal distribution with mean 3668 grams and standard deviation 511 grams. We want to find the percent of babies with weights less than 2500 grams (see graph below). **Step 2: Perform calculations. Show your work.** The standardized score for the boundary value is $z = \dfrac{2500 - 3668}{511} = -2.29$. From Table A, the proportion of *z*-scores below –2.29 is 0.0110. *Using technology:* The command normalcdf(lower: –1000, upper: 2500, μ: 3668, σ: 511) gives an area of 0.0111. **Step 3: Answer the question.** About 1% of babies will be identified as low birth weight.

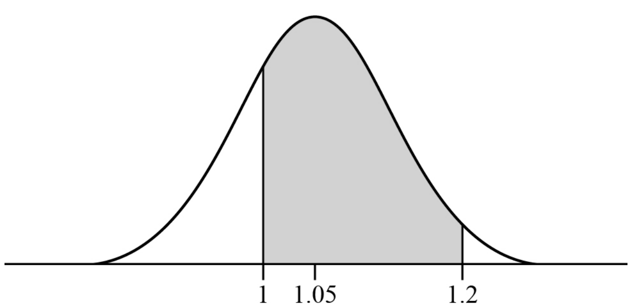


(b) **Step 1: State the distribution and values of interest.** Birth weights follow a Normal distribution with mean 3668 grams and standard deviation 511 grams. The first quartile is the boundary value with

25% of the area to its left. The third quartile is the boundary value with 75% of the area to its left (see graph below). **Step 2: Perform calculations. Show your work.** Look in the body of Table A for a value closest to 0.25. A z-score of –0.67 gives the closest value (0.2514). Solving $-0.67 = \frac{x-3668}{511}$ gives Q_1 = 3325.63. Now, look in the body of Table A for a value closest to 0.75. A z-score of 0.67 gives the closest value (0.7486). Solving $0.67 = \frac{x-3668}{511}$ gives Q_3 = 4010.37. *Using technology:* The command invNorm(area: 0.25, μ: 3668, σ: 511) gives Q_1 = 3323.34. The command invNorm(area: 0.75, μ: 3668, σ: 511) gives Q_3 = 4012.66. **Step 3: Answer the question.** The quartiles are Q_1 = 3323.34 grams and Q_3 = 4012.66 grams.

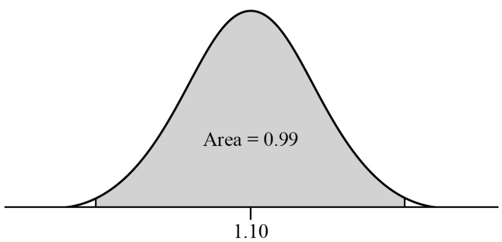


R2.8 (a) **Step 1: State the distribution and values of interest.** The amount of ketchup dispensed follows a Normal distribution with mean 1.05 ounces and standard deviation 0.08 ounces. We want to find the percent of times that the amount of ketchup dispensed will be between 1 and 1.2 ounces (see graph below). **Step 2: Perform calculations. Show your work.** The standardized score for the boundary values are $z = \frac{1.2-1.05}{0.08} = 1.88$ and $z = \frac{1-1.05}{0.08} = -0.63$ From Table A, the proportion of z-scores below z = –0.63 is 0.2643 and the proportion of z-scores below 1.88 is 0.9699. Thus, the proportion of z-scores between –0.63 and 1.88 is 0.9699 – 0.2643 = 0.7056. *Using technology:* The command normalcdf(lower: 1, upper: 1.2, μ: 1.05, σ: 0.08) gives an area of 0.7036. **Step 3: Answer the question.** About 70% of the time the dispenser will put between 1 and 1.2 ounces of ketchup on a burger.



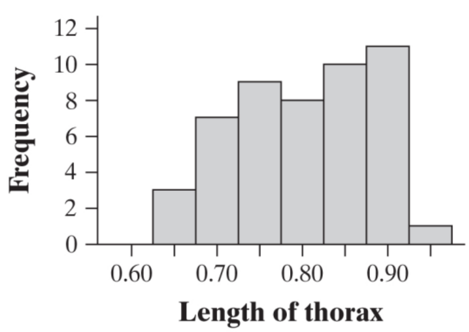
(b) **Step 1: State the distribution and values of interest.** The amount of ketchup dispensed follows a Normal distribution with mean 1.1 and standard deviation σ. We want to find the value of σ that will result in at least 99% of burgers getting between 1 and 1.2 ounces of ketchup (see graph below). **Step 2: Perform calculations. Show your work.** Because the mean of 1.1 is in the middle of the interval from 1 to 1.2, we are looking for the middle 99% of the distribution. This leaves 0.5% in each tail. Look in the body of Table A for a value closest to 0.005. A z-score of –2.58 gives the closest value (0.0049). Solving

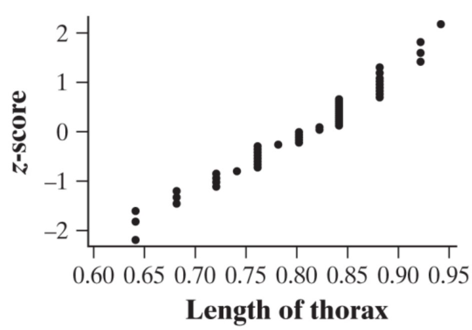
$-2.58 = \dfrac{1-1.1}{\sigma}$ gives $\sigma = 0.039$. *Using technology:* The command invNorm(area: 0.005, μ: 0, σ: 1) gives $z = -2.576$. Solving $-2.576 = \dfrac{1-1.1}{\sigma}$ gives $\sigma = 0.039$. **Step 3: Answer the question.** A standard deviation of at most 0.039 ounces will result in at least 99% of burgers getting between 1 and 1.2 ounces of ketchup.



R2.9 If the distribution is Normal, it must be symmetric about its mean. In particular, the 10th and 90th percentiles must be equal distances above and below the mean, so the mean is $\dfrac{25+475}{2}$ = 250 points. The 10th percentile in a standard Normal distribution is $z = -1.28$. Thus, we solve $-1.28 = \dfrac{25-250}{\sigma}$ and get σ = 175.8. *Using technology*: The command invNorm(area: 0.10, μ : 0, σ : 1) gives $z = -1.282$, so $-1.282 = \dfrac{25-250}{\sigma}$ and get σ = 175.5.

R2.10 A histogram and Normal probability plot are given below. The histogram is roughly symmetric but not very bell-shaped. The Normal probability plot, however, is roughly linear. For these data, the mean is 0.8004 and the standard deviation is 0.0782. 27/49 = 55.1% of the lengths are within 1 standard deviation of the mean (0.7222 to 0.8786). 46/49 = 93.9% of the lengths are within 2 standard deviations of the mean (0.6440 to 0.9568). 49/49 = 100% of the lengths are within 3 standard deviations of the mean (0.5658 to 1.035). Although the percentage within 1 standard deviation of the mean (55.1%) is less than expected (68%), the percentage within 2 and 3 standard deviations match the 68–95–99.7 rule quite well. Based on the graphical and numerical evidence, it is reasonable to say that these data are approximately Normally distributed.





R2.11 The steep, nearly vertical portion at the bottom and the clear bend to the right indicate that the distribution of the data is right-skewed with several outliers. In other words, these measurements are not approximately Normally distributed.